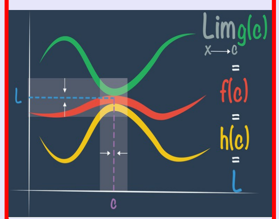
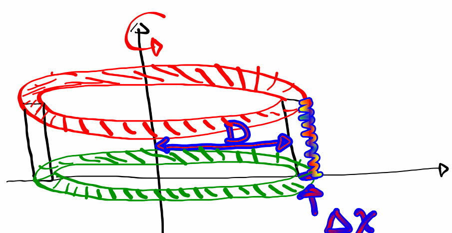
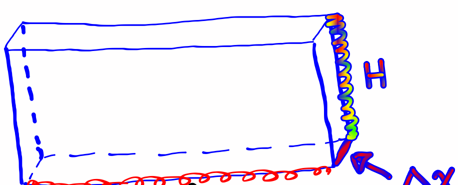
Calculus I

Lecture 55



Feb 19-8:47 AM





Circumference of that circle

$2\pi R$

$2\pi D$

Ref. Rect. is parallel to A.O.R.

Rectangular Box

$V = L\,W\,H$

$= 2\pi D\, \Delta X\, H$

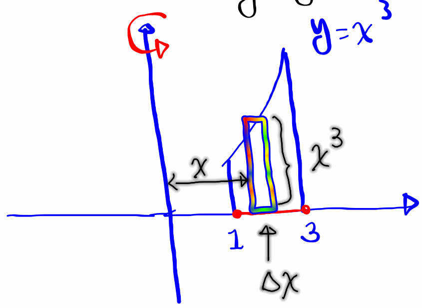
$= 2\pi D\, H\, \Delta X$

Shell Method

$$V = \int_a^b 2\pi\, D\, H\, dx$$

May 23-8:51 AM

Rotate the region enclosed by $y=x^3$, $y=0$, $x=1$, and $x=3$ by y-axis. Find its volume.



Ref. Rect. is parallel to A.O.R.

Shell Method

$$V=\int_1^3 2\pi\, x\, x^3\,dx$$

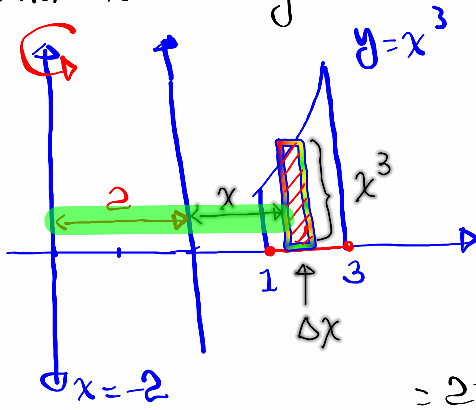
$$=2\pi\int_1^3 x^4\,dx$$

$$=2\pi\cdot\frac{x^5}{5}\Big|_1^3=\frac{2\pi}{5}\left(3^5-1^5\right)$$

$$=\boxed{\frac{484\pi}{5}}$$

May 23-8:57 AM

Rotate the region enclosed by $y=x^3$, $y=0$, $x=1$, and $x=3$ by $x=-2$. Find its volume.



Ref. Rect. is parallel to A.O.R.

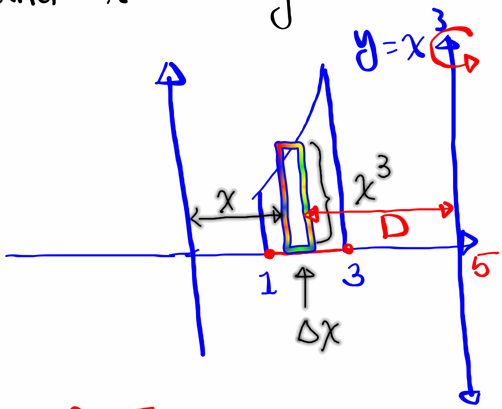
$$V=\int_1^3 2\pi(x+2)\,x^3\,dx$$

$$=2\pi\int_1^3 \left(x^4+2x^3\right)dx$$

$$=2\pi\left[\frac{x^5}{5}+\frac{2x^4}{4}\right]\Bigg|_1^3=\boxed{}$$

May 23-8:57 AM

Rotate the region enclosed by $y=x^3$, $y=0$, $x=1$, and $x=3$ by $x=5$. Find its volume.



Ref. Rect. is parallel to A.O.R.

Shell Method

$$V=\int_1^3 2\pi(5-x)\,x^3\,dx$$

$$=\boxed{\quad}$$

$x+D=5$

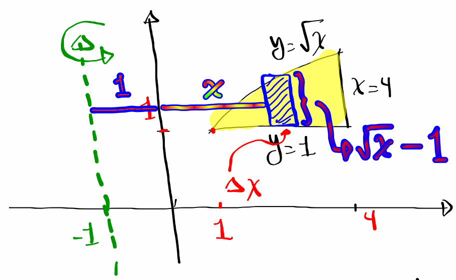
$D=5-x$

May 23-8:57 AM

Draw the region enclosed by $y=\sqrt{x}$, $y=1$, and $x=4$.

Rotate by $x=-1$,

Find the Volume.



Ref. Rect. is parallel to the A.O.R.

$D=x+1$

$H=\sqrt{x}-1$

Shell

$$V=\int_1^4 2\pi(x+1)(\sqrt{x}-1)\,dx$$

$$=2\pi\int_1^4\left[x^{3/2}-x+x^{1/2}-1\right]dx$$

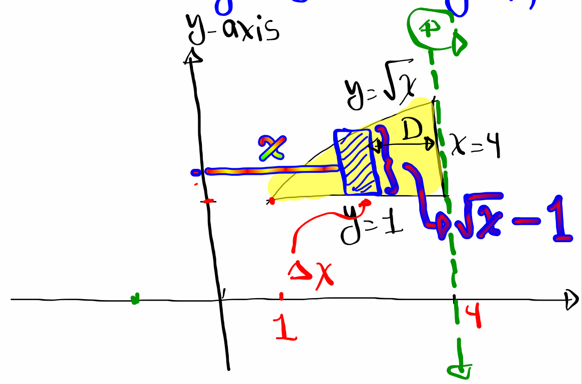
$$=2\pi\left[\frac{x^{5/2}}{5/2}-\frac{x^2}{2}+\frac{x^{3/2}}{3/2}-x\right]\Bigg|_1^4=\boxed{\quad}$$

May 23-9:12 AM

Draw the region enclosed by $y=\sqrt{x}$, $y=1$, and $x=4$.

Rotate by $x=4$,

Find the Volume.



$x + D = 4$

$D = 4 - x$

Ref. Rect. is parallel to the A.O.R.

Shell Method

$$\boxed{V = \int_1^4 2\pi(4-x)(\sqrt{x}-1)\,dx}$$

Set-Up Only

Graph $f(x) = \dfrac{1}{x^2+1}$

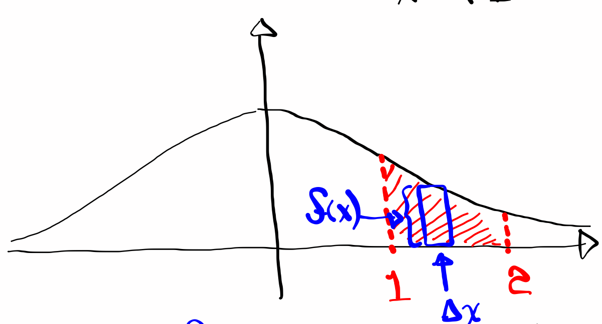
Domain $\to (-\infty, \infty)$

No x-Int

Y-Int $(0,1)$

$\lim\limits_{x\to\pm\infty} f(x) = 0$

Even Function

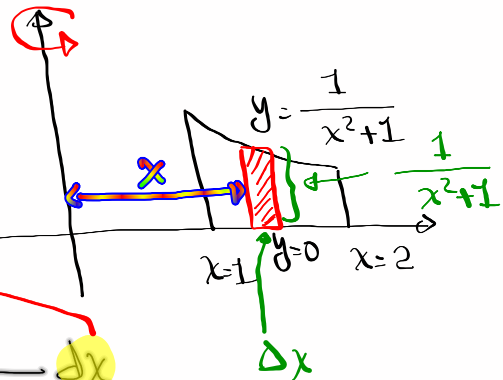


$$\text{Area} = \int_1^2 \frac{1}{x^2+1}\,dx \Rightarrow$$ Calc. II Table of integrations

what if we rotate this region by y-axis, Find its volume.

Ref. Rect. is parallel to A.O.R.

Shell Method



$$V = \int_1^2 2\pi \cdot x \cdot \frac{1}{x^2+1}\,dx$$

$u = x^2+1$

$du = 2x\,dx$

$x=1 \to u=2$

$x=2 \to u=5$

$$= \pi \int_2^5 \frac{1}{u}\,du = \pi \ln u \Big|_2^5$$

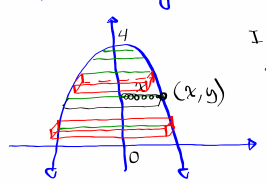
$$\int \frac{1}{x}\,dx = \ln|x| + C \qquad = \pi\left[\ln 5 - \ln 2\right] = \boxed{\pi \ln \frac{5}{2}}$$

May 23-9:32 AM

How to Find Volume of a Solid without rotation:

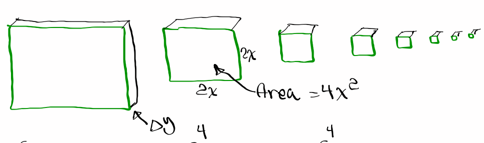
Consider the region enclosed by $y = 4 - x^2$ and $y = 0$

$x^2 = 4 - y$



I like to place Square shapes ⊥ Y-axis

Once we attach these Squares, we get a Solid



$$V = \int A(\text{Base})\,dy = \int_0^4 4x^2\,dy = 4\int_0^4 (4-y)\,dy$$

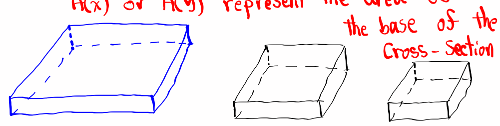
$$= 4\left[4y - \frac{y^2}{2}\right]\Bigg|_0^4 = 4\left[4(4) - \frac{4^2}{2}\right] = 4(16-8)$$

$$= 4(8)$$

$$= 32$$

$V = \int_a^b A(x)\,dx$ or $V = \int_c^d A(y)\,dy$

$A(x)$ or $A(y)$ represent the area of the base of the Cross-Section



May 23-9:39 AM